

PROPOSITIONS BASED APPLICATIONS OF DIGITAL CIRCUITS

Propositions based Applications of Digital Circuits are those in which a digital circuit is planned according to certain given criteria. The inputs/outputs are propositions and are plotted in a truth table. The truth table generates the corresponding Boolean expression. It is plotted in a K-Map and is simplified (if possible). The final expression is then used to create the combinational digital circuit.

The following examples show how to implement the above :

Example 1 : A family decides to create a Voting Machine that will help them to take decision. There were four members in the family; Father (F), Mother (M), Son (S) and Daughter (D). Any proposal gets passed only if any one of the following conditions is true

- Both parents say yes
- Both children and Father say yes
- Both children and Mother say yes

Inputs are :

F – Father voting Yes

M – Mother voting Yes

S – Son voting Yes

D – Daughter voting Yes

Output is : P : The proposal is passed. [1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above. Write the Sum Of Product expression for P (F, M, S, D).
- Reduce P (F, M, S, D) using Karnaugh's map.
- Draw the logic gate diagram for the reduced expression for P (F, M, S, D) using elementary gates. You may use gates with two or more inputs.

Solution :

- Planning the truth table :** For four inputs, there will be four input columns in the truth table. For one output there will be one output column. All possible combinations of the input set has to be plotted and for each criteria of positive output, the output column term has to be made 1. Finally for the non-happening criteria, the output column term has to be made 0.

II. The Truth Table

F	M	S	D	P	Term Num
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	0	2
0	0	1	1	0	3
0	1	0	0	0	4
0	1	0	1	0	5
0	1	1	0	0	6
0	1	1	1	1	7
1	0	0	0	0	8
1	0	0	1	0	9
1	0	1	0	0	10
1	0	1	1	1	11
1	1	0	0	1	12
1	1	0	1	1	13
1	1	1	0	1	14
1	1	1	1	1	15

III. The cardinal form of P is

$$P(F, M, S, D) = \Sigma(7, 11, 12, 13, 14, 15)$$

IV. The Karnaugh's Map and its reduction using groups

F.M \ S.D	S'.D' [0 0]	S'.D [0 1]	S.D [1 1]	S.D' [1 0]
F'.M' [0 0]	0 0	0 1	0 3	0 2
F'.M [0 1]	0 4	0 5	1 7	0 6
F.M [1 1]	1 12	1 13	1 15	1 14
F.M' [1 0]	0 8	0 9	1 11	0 10

IV. The Reduced Expression

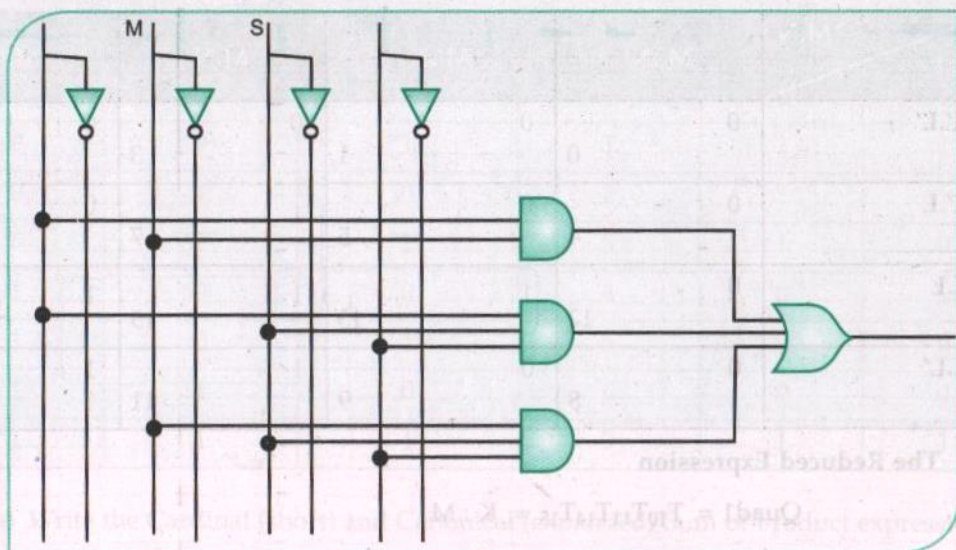
$$\text{Quad1} = T_{12}T_{13}T_{14}T_{15} = \text{F.M}$$

$$\text{Pair1} = T_7T_{15} = \text{S.D.M}$$

$$\text{Pair2} = T_{11}T_{15} = \text{S.D.F}$$

$$\text{Final Reduced Expression } P = \text{F.M} + \text{S.D.M} + \text{S.D.F}$$

V. The Logic Gate Diagram



Example 2 : Four coins K, L, M, N are tossed together (1 for head , 0 for tail). A win W is said to occur if

- three or more coins show head on fall
- coin K and coin M show head on fall

Inputs are :

K : Coin 1 showing head.

L : Coin 2 showing head.

M : Coin 3 showing head.

N : Coin 4 showing head.

Output is :

W : A Win occurring.

[1 indicates yes & 0 indicates no]

Solution :

I. **Planning the truth table :** There are 4 input lines (K, L, M, N) and 1 output line (W).

II. The Truth Table

K	L	M	N	W	Term Num
0	0	0	0	0	0
0	0	0	1	0	1
0	0	1	0	0	2
0	0	1	1	0	3
0	1	0	0	0	4
0	1	0	1	0	5
0	1	1	0	0	6
0	1	1	1	1	7
1	0	0	0	0	8
1	0	0	1	0	9
1	0	1	0	1	10
1	0	1	1	1	11
1	1	0	0	0	12
1	1	0	1	1	13
1	1	1	0	1	14
1	1	1	1	1	15

III. The cardinal form of W is

$$W (K, L, M, N) = \Sigma (7, 10, 11, 13, 14, 15)$$

IV. The Karnaugh's Map and its reduction using groups

K.L \ M.N	M'.N'	M'.N	M.N	M.N'
K'.L'	0	0	0	0
K'.L	0	0	1	0
K.L	0	1	1	1
K.L'	0	0	1	1

The Reduced Expression

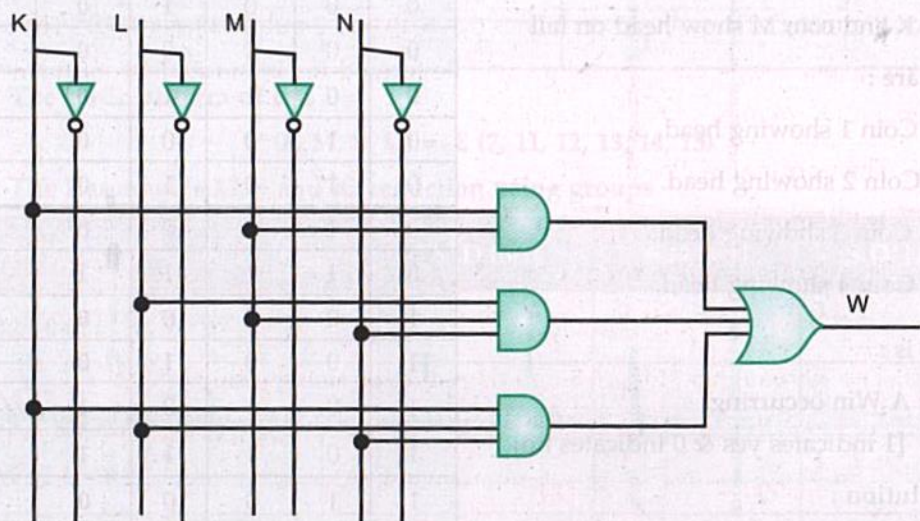
$$\text{Quad1} = T_{10}T_{11}T_{14}T_{15} = K \cdot M$$

$$\text{Pair1} = T_7T_{15} = L \cdot M \cdot N$$

$$\text{Pair2} = T_{13}T_{15} = K \cdot L \cdot N$$

Final Reduced Expression $W = K \cdot M + L \cdot M \cdot N + K \cdot L \cdot N$

V. The Logic Gate Diagram



Example 3 : A Full Subtractor is a combinational circuit that performs the subtraction between two bits, taking into account that 1 may have been borrowed by a lower significant stage. Given below is the truth table for a Full Subtractor. It has three inputs M (Minuend), S (Subtrahend), P (Previous Borrow). It has two outputs B (Borrow) and D (Difference).

Inputs			Term Num	Outputs	
M	S	P		B	D
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	2	1	1
0	1	1	3	1	0
1	0	0	4	0	1
1	0	1	5	0	0
1	1	0	6	0	0
1	1	1	7	1	1

(a) Write the Cardinal (short) and Canonical (expanded) Sum of Product expression for both the outputs B and D.

(b) Use Karnaugh's map to reduce B and D if possible.

(c) Draw the Logic Gate Diagram of the reduced expressions of B and D.

Solution :

I. Planning : There are 3 input lines (M, S, P) and 2 output lines (B, D) . The truth table has been given. 2 separate expressions will be formed for B and D. Both of them will need two separate Karnaugh's Map for reduction.

II. The Cardinal SOP form of B is $B (M, S, P) = \Sigma(1, 2, 3, 7)$

The Canonical SOP form of B is $M'.S'.P + M'.S.P' + M'.S.P + M.S.P$

The Cardinal SOP form of D is $D (M, S, P) = \Sigma (1, 2, 4, 7)$

The Canonical SOP form of D is $M'.S'.P + M'.S.P' + M.S'.P' + M.S.P$

III. The Karnaugh's Map of B and its reduction using groups

M \ S, P	S'.P'	S'.P	S.P	S.P'
M'	0	1	1	1
M	0	0	1	0

The Reduced Expression of B

Pair1 = $T_1 T_3 = M'.P$

Pair2 = $T_2 T_3 = M'.S$

Pair3 = $T_3 T_7 = S.P$

Final Reduced Expression of $B = M'.P + M'.S + S.P$

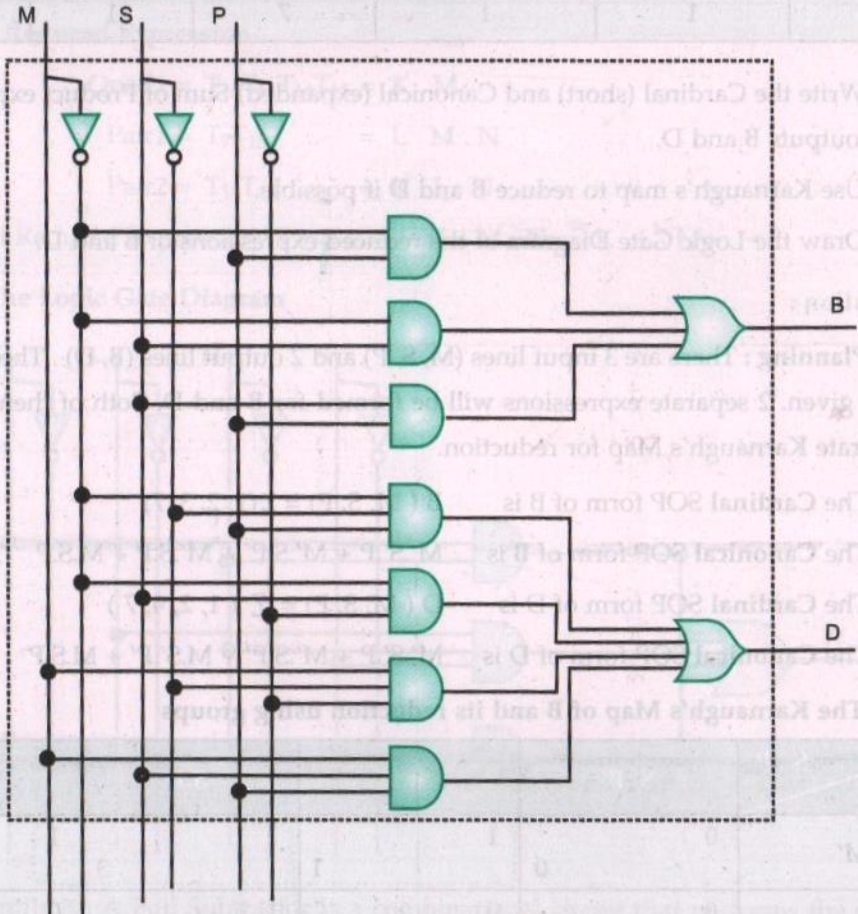
The Karnaugh's Map of D and its reduction using groups

M \ S . P	S . P	S'.P'	S'.P	S.P	S.P'
M'	0	1	0	1	2
		0	1	3	
M	1	0	1	0	6
		4	5	7	

The Karnaugh's map indicates that the expression cannot be reduced as no groups are getting formed.

The expression of $D = M'.S'.P + M'.S.P' + M.S'.P' + M.S.P$

IV. The Logic Gate Diagram of a Subtractor



Example 4 : Mr B wanted to design a machine for taking decision on whether or not he should go for morning walk on a particular day. He goes for a morning walk if :

- he has to go for check-up and he has night shift duty.
- there is someone at home and the season is not summer.
- there is no one at home but he has to go for check-up.

Inputs are :

C : He has to go for check-up.

D : He does not have night shift duty.

H : There is no one at home.

S : The season is summer.

Output is :

W : Mr B goes for a walk.

[1 indicates yes & 0 indicates no]

(a) Draw the truth table for the inputs and outputs given above. Write the Sum Of Product expression for W (C, D, H, S).

(b) Reduce W (C, D, H, S) using Karnaugh's map.

(c) Draw the logic gate diagram for the reduced expression for W (C, D, H, S) using elementary gates. You may use gates with two or more inputs.

Solution :

I. Planning the truth table : Before plotting the conditions in the truth table, they can be analysed to streamline the output W.

Condition 1 : he has to go for check-up and he has night shift duty.

C = 1; D = 0; H and S hold both 0, 1.

Term Numbers that become high are (8, 9, 10, 11)

Condition 2 : there is someone at home and the season is not summer.

H = 0; S = 0 C, D hold both 0, 1.

Term Numbers that become high are (0, 4, 8, 12)

Condition 3 : there is no one at home but he has to go for check-up.

H = 1; C = 1 S, D hold both 0, 1.

Term Numbers that become high are (10, 11, 14, 15)

These terms will be plotted in the truth table. Once a term is set to high, it remains high for further cases.

II. The Truth Table :

C	D	H	S	W	Term Num
0	0	0	0	1	0
0	0	0	1	0	1
0	0	1	0	0	2
0	0	1	1	0	3
0	1	0	0	1	4
0	1	0	1	0	5
0	1	1	0	0	6
0	1	1	1	0	7
1	0	0	0	1	8
1	0	0	1	1	9
1	0	1	0	1	10
1	0	1	1	1	11
1	1	0	0	1	12
1	1	0	1	0	13
1	1	1	0	1	14
1	1	1	1	1	15

III. The cardinal form of W is

$$W(C, D, H, S) = \Sigma(0, 4, 8, 9, 10, 11, 12, 14, 15)$$

IV. The Karnaugh's Map is shown below

C.D \ H.S	H.S	H'.S'	H'.S	H.S	H.S'
C'.D'	1	0	0	0	0
C'.D	1	4	5	7	6
C.D	1	12	13	15	14
C.D'	1	8	9	11	10

Student's exercise to write the reduced expression from the K-Map and draw the Logic Gate Diagram.

Exercise

1. Jerry accepts an invitation if one of the following conditions are true :

- Tom has invited and he is fit.
- Tom has invited and the conference is in Taj Bengal.
- The conference is in Taj Bengal and the car is available.

Inputs are :

- T – Tom has invited.
- B – The conference is in Taj Bengal.
- C – The car is available.
- F – He is fit.

Output is :

- A – He accepts invitation.

[1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above.
- Write the cardinal Sum of Product expression for A (T, B, C, F).
- Reduce A (T, B, C, F) using Karnaugh's map and write the reduced expression.
- Draw the logic gate diagram for the reduced expression for A (T, B, C, F) using elementary gates. You may use gates with two or more inputs.

2. Venezia bank grants loan to an applicant if :

- the applicant is an Indian above 18 years old, cannot produce surety of loan but the loan is for cottage industry.
- the applicant is an Indian above 18 years old and can produce surety of loan.
- the applicant is an Indian below 18 years of age, who can produce surety of loan and the loan scheme is for cottage industry.
- the applicant is not an Indian who is above 18 years old and can produce surety of loan.

Inputs are :

- Y : The applicant is above 18 years old.
- N : The applicant is an Indian.
- S : The applicant can produce surety of loan.
- C : Loan scheme is for cottage industry.

Output is :

G : Granting of loan [1 indicates yes & 0 indicates no]

- (a) Draw the truth table for the inputs and outputs given above.
- (b) Write the cardinal Sum Of Product expression for G (Y, N, S, C).
- (c) Reduce G (Y, N, S, C) using Karnaugh's map.
- (d) Draw the logic gate diagram for the reduced expression for G (Y, N, S, C) using elementary gates. You may use gates with two or more inputs.

3. In a function Special dress circle invitation cards is to be issued to a person if :

- The person is a local resident and of age more than 60 yrs.
- The person has sponsored >50000 but is not a local resident.
- The person has not sponsored >50000 but is recommended by a performer.

Inputs are :

S : The person has sponsored > 50000.

L : The person is a local resident.

R : The person is recommended by a performer.

A : The person is >60 years of age. [1 indicates yes, 0 indicates no]

Output is :

V : Invitation card to be issued. [1 indicates yes & 0 indicates no]

- (a) Draw the truth table for the inputs and outputs given above.
- (b) Write the cardinal Sum Of Product expression for V(S, L, R, A).
- (c) Reduce V(S, L, R, A) using Karnaugh's map.
- (d) Draw the logic gate diagram for the reduced expression for V(S, L, R, A) using elementary gates. You may use gates with two or more inputs.

4. A vehicle is allowed to pass "Toll Bridge Number 7" only if it satisfies any one of the following conditions :

- The vehicle is blue in color, belongs to an MLA and has red blinkers.
- The vehicle is not blue but has red blinkers and is carrying gifts of a ceremony.
- The vehicle belongs to an MLA and has red blinkers.
- The vehicle is blue in color, is not carrying gifts of a ceremony and has red blinkers.

Inputs are :

B : The vehicle is blue in color.

R : The vehicle has red blinker.

G : The vehicle is carrying gifts for a ceremony.

M : The vehicle belongs to an MLA.

Output is :

T : The vehicle is allowed to pass "Toll Bridge Number 7". [1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above.
- Write the cardinal Sum Of Product expression for T (B, R, G, M).
- Reduce T (B, R, G, M) using Karnaugh's map.
- Draw the logic gate diagram for the reduced expression for T (B, R, G, M) using elementary gates. You may use gates with two or more inputs.

5. In a Tanayaka Higher Secondary School, the School Board decided that a student should be given homework if any of the following conditions are true :

- The student was absent for 2 tests and got less than 60 % marks in test average.
- The student got more than equal to 60 % marks in test average but did not come to school in proper uniform and took part in nature quiz.
- The student was absent for 2 tests and did not come to school in proper uniform.
- The student less than 60 % marks in test average and did not take part in nature quiz.

Inputs are :

B : The student was absent for 2 tests.

M : The student got less than 60 % marks in test average.

Q : The student did not take part in nature quiz.

U : The student did not come to school in proper uniform.

Output is :

H : The student will be given homework. [1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above.
- Write the cardinal Sum Of Product expression for H (B, M, Q, U).
- Reduce H (B, M, Q, U) using Karnaugh's map.
- Draw the logic gate diagram for the reduced expression for H (B, M, Q, U) using elementary gates. You may use gates with two or more inputs.

6. In a Home Delivery Store, it was observed that the sales were more if any one of the following conditions were true :

- There were rains and the time was after 8 pm.
- The season was summer and the year-end sale was on.
- The year-end sale was not on but there were rains.
- The time was not after 8 pm and the year-end sale was not on.

Inputs are :

R : There were rains.

S : The season was summer.

T : The time was after 8 pm.

Y : The year-end sale was not on.

Output is :

M : The sales were more.

[1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above.
- Write the cardinal Sum Of Product expression for $M (R, S, T, Y)$.
- Reduce $M (R, S, T, Y)$ using Karnaugh's map.
- Draw the logic gate diagram for the reduced expression for $M (R, S, T, Y)$ using elementary gates. You may use gates with two or more inputs.

7. A Company's Negotiation of a financial deal would turn into a Deadlock if any one of the following occurred :

- a high profile customer wanted a discount of the lowest rate which can't be given.
- the vendor imposes tax on a not so high profile customer when the rates are not lowest.
- the rates are not lowest to a high profile customer.
- the vendor can't give discount on the lowest rate to a not so high profile customer.

The Inputs are :

H : the customer is high profile.

L : the rates are lowest.

T : the vendor imposes tax.

D : the vendor can't give discount.

Output is :

N : the negotiation turns into a deadlock.

[1 indicates yes & 0 indicates no]

- Draw the truth table for the inputs and outputs given above.
- Write the cardinal Sum Of Product expression for $N (H L T D)$.
- Reduce $N (H L T D)$ using Karnaugh's map.

